Errata to Class Field Theory

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Page 70, proof of Lemma 1.37: I think the splitting $M_* \approx \text{Ker}(\varphi) \oplus \text{Ker}(1 - \varphi)$ should be $M_* \approx \text{Ker}(s) \oplus \text{im}(\varphi)$ (where $\varphi$ is the map $m \mapsto \varphi_m$, discussed at the beginning of the proof).

Page 71, Proposition 1.39(b): this statement should be understood taking into account the canonical isomorphism $M \otimes N \to N \otimes M$.

Page 84: that this cohomology is only defined for $r \geq 0$ might be worth emphasizing.

Page 84: I think the group $H_1$ might not be small enough for the present purpose. What we have to do instead is as follows: for any point in $G^r$, take a translate of an open normal subgroup containing it and fully contained in a fiber of $\varphi$. By compactness of $G^r$ we may replace this open cover by a finite one. Then let $H_1$ be the intersection of all subgroups whose translates have been used.

Page 97: in the second sentence of The invariant map section, we actually have to use $H^2(G, U_L) = H^3(G, U_L) = 0$.

Page 100: in the third displayed equation, $\delta \hat{f}$ at the beginning should be $d \hat{f}$.

Page 102, Theorem 2.1: $L/K$ should be assumed Galois. Alternatively, the remark from the bottom of page 103 should be moved here.

Page 102, Lemma 2.3: at the end of the statement, “all $r > 0$” should be “for all $r > 0$”.

Page 104, proof of Lemma 2.7: in line -5, a reference to (3.15) should be a reference to (30). Also, doesn’t this mean equality (30) is proven? I’m not sure I understand the footnote in this case.

Page 105: around the middle of the page, a relation $\text{Inf}(x \cup u_{E/K}) = [L : E]\text{Inf}(x) \cup u_{L/K}$ is stated. However, firstly, the relevant relation between inflation and cup products is not stated in Proposition II 1.39, nor anywhere else. Furthermore, the footnote on page 77 claims Inf does not extend to negatively indexed Tate groups, so a remark should be made that this is only for $r \geq 1$.

Page 112, proof of Theorem 4.4: in the last line of the proof, $1 \mod \mathbb{Z}$ should be $1 \mod n$.

Pages 113/114: characteristic zero assumption is also crucially used in Step 3 to use the theory of Hilbert symbol. In light of I 1.3, this assumption isn’t really needed in regard to openness of finite subgroups.

Page 115: there is a sudden change from multiplicative to additive notation.

Page 154, line -5: the term ”conductor” is used despite only being defined on the next page.
Page 157, Theorem 3.16: in the statement, \( L/K \) is assumed Galois, but in the following example it’s not Galois. It’s not clear to me how to deduce the non-Galois version from the Galois version, since the Galois closure may contain a larger abelian extension.

Page 160, Theorem 3.23: the last part of the statement should say that the density is \( 1/(G : 1) \), not \( (G : 1) \).

Page 166, line 10: \( U_v \) was already defined to denote \( \mathcal{O}_v^{\times} \).

Page 171, Example 4.10: the impact of \( a_\infty \) on the Artin map is not considered. Further, in the last part of the proof, \( c \) should be taken positive.

Page 173: it might be worth repeating the definition of the local Artin map at infinite primes, or at least referencing I 1.6.

Page 203, Proposition 2.8: this result is already shown in V 4.13, upon observation that each \( W_m \) is open.

Page 208, proof of Proposition 4.7: it should be noted that \( H \) is a normal subgroup. For that we may want to replace \( T \) by a larger \( G \)-stable set. Additionally, in the displayed equation, \( \mathfrak{F} \) on the left should be replaced with \( \mathfrak{F} \cap E \).

Page 212, proof of Lemma 6.2: in line 9, we should say “form a basis of \( \text{Gal}(M/L) \)”, not “form a basis of \( \text{Gal}(M/K) \)”.

Page 216, proof of Proposition 7.2: in line 6, \( H^2(L/K) \) probably should be \( H^2(/K) \). In line 7, the statement would be made less trivial if \( m \) was assumed nonzero.

Page 218, proof of Lemma 8.4: in the second diagram, \( \mathbb{V}_K \) should be \( \mathbb{I}_K \). In the sentence that follows, it should be said that \( K^{\times} \) is mapped into \( K^{\times} \), and I don’t see what “previous one” refers to here.

Page 219, proof of Lemma 8.5: in last line, \( \delta_X \) should be \( \delta_X \).

Page 219, proof of Lemma 8.6: notation \( H^2(/K) \) introduced at the beginning of section 7 becomes \( H^2(K) \).

Page 220, Proposition 9.2: since \( L/K \) in the proof is cyclic, it appears that the proof only uses the First Inequality (specifically Lemma 4.5), not the full Reciprocity Law. Regardless, it appears that this Proposition is not used anywhere.

Page 237, proof of Lemma 4.1: in line 3 there is a reference missing, presumably to VII 4.8.