COMMENTS ON MILNE, INTRODUCTION TO SHIMURA VARIETIES

These are comments on the October 23, 2004 version of Milne, *Introduction to Shimura varieties*, available at `http://www.jmilne.org/math/xnotes/svi.html` . The comments here are in addition to the errata listed at that website. The following people have contributed to this list: Yuzhou Gu, Naoki Imai, Bjorn Poonen, Jun Su, Isabel Vogt, Lynnelle Ye.

- p12: It would be better if the definition of **adjoint** came before it is used in the table.
- p13, footnote 12: Define e_m here. (The definition appears only later, on p14.)
- p15, line before Proposition 1.7: It has not been previously mentioned that $\operatorname{Hol}(M)^+$ is an adjoint group. Is the intended argument that one should combine the table on p12 with Proposition 1.6?
- p15, Proposition 1.7: Missing letter in "stabilizer".
- p16, definition of **canonical tensor**: What does "canonically derived" mean?
- p17, first display in Example 1.15: Change $\theta\left(\left(\begin{smallmatrix} a & b \\ c & d \end{smallmatrix}\right)\right)$ to $\theta\left(\overline{\left(\begin{smallmatrix} a & b \\ c & d \end{smallmatrix}\right)}\right)$.
- p17, Theorem 1.16: It would be clearer to say "any two are conjugate by $\operatorname{ad} g$ for some $g \in G(\mathbb{R})$". That is, if C and C' are two Cartan involutions, then $C' = (\operatorname{ad} g) \circ C' \circ (\operatorname{ad} g)^{-1}$ for some $g \in G(\mathbb{R})$.
- p17, last sentence: It is not clear what is being claimed in the last half of this sentence, or what the logic is. First, there are two possible notions of "real form" of $G_{\mathbb{C}}$: one is an algebraic group H over $\mathbb{R}$ such that $H_{\mathbb{C}} \simeq G_{\mathbb{C}}$, and the other is an algebraic group H over $\mathbb{R}$ *equipped with an isomorphism* $H_{\mathbb{C}} \simeq G_{\mathbb{C}}$; it seems that the latter notion is being used here. In other words, it seems that if two anti-holomorphic involutions of $G(\mathbb{C})$ are conjugate but different, then they are considered to give different real forms. A map

$$\{\text{Cartan involutions of } G\} \to \{\text{compact forms of } G_{\mathbb{C}}\}$$

has been defined. But it does not seem to be surjective since one can conjugate by elements of $G(\mathbb{C})$ on the right but only by elements of $G(\mathbb{R})$ on the left. For a similar reason (conjugation by elements of $G(\mathbb{C})$), in the second sentence of Example 1.17(b), it seems to be not true that all real forms arise from involutions θ of G defined over $\mathbb{R}$ (this statement was not actually claimed, but readers might read this between the lines as being the explanation for the last sentence). In any case, it has not been explained why any map like the map displayed above is injective or surjective, so it seems wrong to say "we see that" in the last sentence.
- p18, middle: Use `\DeclareMathOperator{\ad}{ad}` in the preamble and then `\ad` in the text to get suitable spacing in expressions like $\operatorname{ad} C$.
- p18, first line of proof of Proposition 1.20: It would be more informative to say "extends to a unique" instead of "defines a".
- p18, after (11): It would help to have an additional line rewriting this as

$$(\varphi_C)'(gu, ((\operatorname{ad} C)\overline{g})v) = (\varphi_C)'(u, v).$$

- p19, line 1: Since $\varphi'_C = (\varphi')_C$ has not been defined, it would be better to write $(\varphi_C)'$.

Date: April 11, 2016.

- p20, line 2: At the end of part (a), it would be clearer to write $\mathrm{ad} \circ u_p$ instead of u_p.
- p20: To explain the action of U_1 on the spaces in the first display, mention before this that in the isomorphism $G(\mathbb{R})^+/K_p \cong D$, the group U_1 acts compatibly on $G(\mathbb{R})^+ = \mathrm{Hol}(D)^+$ and K_p (by conjugation) and on D (via u_p).
- p20, sentence containing the first display: To make sense of multiplication by a complex number acting on an $\mathbb{R}$-vector space, after "as multiplication by z" insert "(when T_pD is viewed as a $\mathbb{C}$-vector space)".
- p20, same sentence: To explain why $u_p(z)$ acts trivially on $\mathrm{Lie}(K_p)$, mention that conjugation by $u_p(z)$ on K_p is the identity: if $k \in K_p$, then $u_p(z)ku_p(z)^{-1} = k$ by Proposition 1.14 with $p = p'$ and $a := dk$.
- p20, second display: It would help to say that since K_u is the centralizer of u in $G(\mathbb{R})^+$, we have $\mathrm{Lie}(K_u) = \mathrm{Lie}(G)^0$.
- Corollary 1.22: The word "nontrivial" must be deleted to make this true! If G is not $\{1\}$, then the nontriviality of u is implied by (c). But if $G = \{1\}$ (i.e., if D is a point), then u needs to be trivial.
- p21, second sentence: To avoid confusion of G and $G_{\mathbb{C}}$ here, it would be better to write "which is simple because G is an inner form of the compact form of $G_{\mathbb{C}}$".
- p21, Proposition 1.24: Part (a) should say "a simple adjoint group H" so that H is defined later in the sentence.
- p21, second sentence of the proof of Proposition 1.24: Why does u take values in $H(\mathbb{R})$? To ensure this, change "the unique compact real form of G" in the first sentence of the proof to "a maximal compact subgroup of $G(\mathbb{C})$ containing $\mu(U_1)$".
- p21, sentence after the proof of Proposition 1.24: Insert "over" before "$\mathbb{C}$".
- p21, next line: There is a stray comma after "Recall".
- p22, "It should be noted that not every simple real algebraic group arises as the automorphism group of a hermitian symmetric domain. For example, PGL_n arises in this way only for $n = 2$." An easier example is any nontrivial compact simple adjoint real group.
- p22, line 4: One could replace

  "For example, PGL_n arises in this way only for $n = 2$."

  by

  "If G arises, and $u = u_p$ is as in Theorem 1.21, then Theorem 1.21 and its proof show that $u(U_1)$ is contained in the center of the maximal compact subgroup K_u of $G(\mathbb{R})^+$ given by the Cartan involution $\mathrm{ad}(u(-1))$; if, moreover, G is nontrivial, so u is nontrivial too, then any maximal compact subgroup (they are all conjugate) of $G(\mathbb{R})^+$ should have nondiscrete centre, but this is not so for $G = \mathrm{PGL}_n$ for $n > 2$."

  to justify this.
- p24, just before (15): The comma should be replaced by the word "is".
- p25, second to last sentences before "Hodge structures": Change F to F_0 (three times). Or change every F_0 to F.
- p26, last sentence of the penultimate paragraph: The wording was confusing at first: it makes it sound as if the two descriptions are parallel, but they are not: the isomorphism $S_{\mathbb{C}} \to \mathbb{G}_m \times \mathbb{G}_m$ is described by giving the *composition* of $S(\mathbb{R}) \to S(\mathbb{C})$ with its induced map on $\mathbb{C}$-points, while $w\colon \mathbb{G}_m \to \mathbb{S}$ is described by giving it on real points. Also, the first two $\mathbb{G}_m$'s are $\mathbb{G}_{m\mathbb{C}}$ and the last one is $\mathbb{G}_{m\mathbb{R}}$.

- p27, (21): r^n should be $r^n v$.
- p28, first display: $V^{r',s'}$ should be $W^{r',s'}$.
- p28, Morphisms of hodge structures: In the first sentence of this section, "Hodge" is capitalized. This also happens twice in Aside 2.16 on p31.
- p28, last display: To make the argument clearer, insert the expression $\psi(u, (-1)^n v)$ between the last two.
- p29, line 6: It would help to reword to clarify that "These conditions" is not referring to only the two displayed conditions.
- p29, definition of variation of hodge structures: It would help to add some motivation for this definition here.
- p30, line 4: $G_{d(p,q)}(V)$ is a variety over $\mathbb{R}$, so probably this should be $G_{d(p,q)}(V)(\mathbb{C})$. Similar problems appear throughout p30 and p31 almost every time that a Grassmannian appears.
- p30, Theorem 2.14(a): Connected components are nonempty by definition, so there is no need to assume that S^+ is nonempty.
- p30, second sentence of proof of Theorem 2.14: x should be s.
- p30, three lines after (25): Remove the multiplication dots to match the previous line.
- p30, two lines before (26): There is not a homomorphism $G \xrightarrow{\mathrm{Ad}} \mathfrak{g}$. Instead G *acts* on $\mathfrak{g}$ and $\mathrm{End}(V)$.
- p30, line before (26): There is a stray comma after "In the diagram".
- p30, (26): Why is $\mathrm{End}(V(\mathbb{C}))/F^0 \cong T_{h_0} G_{\mathbf{d}}(V)$?
- p30, (26): The period at the end should be a comma.
- p31, line 6: Theorem 2.1 should be Proposition 2.1.
- p31, line 7: Change "set of tensors T" to "set of tensors $T \ni t_o$".
- p31, line 8: It would help to insert G between U_1 and $\mathrm{GL}(V)$.
- p34, end of Example 3.4: A period is missing.
- p34, sentence defining **neat**: After "α does not have finite order" add "or $\alpha = 1_V$".
- p35, proof of Proposition 3.6: The open subset U needs to be chosen more carefully since otherwise it is not true as claimed that "its complement intersects Γ intersects Γ in $\{1_\Gamma\}$". It could be that $1_\Gamma = \alpha(g)$ for some $g \in G(\mathbb{R})^+ \setminus U$ lying outside Γ_0.
- p35, next sentence, "we can apply (3.3a)": Perhaps add "to G and Γ_0" for clarity.
- p35, Remark 3.7: subgroup should be subgroups.
- p35, end of first paragraph of Remark 3.7: This is not correct as stated. Should it be "countably many arithmetic subgroups up to $\mathrm{PGL}_2(\mathbb{R})^+$-conjugacy"?
- p36, Remark 3.9: What is called a "complex space" is more commonly called a "complex analytic space".
- p39, Remark 3.13, second part: The first sentence is missing a verb. Maybe change = to "equals"?
- p39, Remark 3.13: The third part should be (c).
- p39, Remark 3.13, third part: "prove" should be "proof".
- p39, Theorem 3.14: The "an" should be in an upright font, not italicized, in order to match the earlier usage.
- p39, last paragraph: "According" should be "According to".
- p39, last paragraph: Hironaka's Fields Medal could have mentioned the first time the theorem was mentioned, on p37.
- p41, Theorem 3.21: What kind of automorphisms? Biholomorphic automorphisms?

- p41, first sentence of the proof of Theorem 3.21: Delete “a” before “torsion free”.
- p41, just after display, “The corollary implies”: To explain why the hypothesis of Corollary 3.20 (i.e., the hypothesis of Theorem 3.19) holds here, it seems that you need Lemma 4.7(a)⇔(c); it would have helped to have this in Chapter 1.
- p41, same sentence: It seems that the argument is “$\mathrm{Aut}(\Gamma)$ is countable, so N is countable”. But it has not been explained why $\mathrm{Aut}(\Gamma)$ is countable. (It is not enough to say that Γ is countable.)
- p42, bottom (excluding footnotes): It is probably more common to use a comma instead of a colon in the notation for a restricted topological product. Also, this would avoid the conflict with your later use of the colon to denote the index of a subgroup.
- p42, footnote 30: If $C(G)$ is nontrivial, might the claim “$C(G')$ contains a subgroup isomorphic to $N(\mathbb{A}_f)/N(\mathbb{Q})$” be wrong? In any case, this claim is not clear. It would be better to say “because $\ker(\hat{\pi}) \to \ker(\bar{\pi})$ is not surjective, $C(G) \to C(G')$ is not surjective, so G' has a noncongruence subgroup”.
- p43, line 1: The second ℓ should be $\mathbb{Z}$ or $\mathbb{Q}$.
- p43, display with $V(\mathbb{A}_f)$: It is not clear whether this is intended to be a definition or a proposition. It could be that you are *defining* $V(\mathbb{A}_f)$ as this restricted topological product. Or it could be that the point of view is that $V(\mathbb{A}_f)$ has already been defined as $V(R)$ in the first display of p43 with $R = \mathbb{A}_f$, and that now it is being claimed that the natural map from the already-defined $V(\mathbb{A}_f)$ to the restricted topological product is a bijection.
- p44, SU1: Change “defined by u” to “defined by $\mathrm{ad} \circ u$” for clarity.
- p44, SU2: G^{ad} should be $G^{\mathrm{ad}}_{\mathbb{R}}$.
- p45, Lemma 4.7: This is only about adjoint real Lie groups, so, as mentioned above in the comments on p41, it would be better to have this in Chapter 1. Perhaps it could go just before Theorem 1.21.
- p45, proof of Lemma 4.7: In the proof of (a)⇔(c), the $\iff$ labeled with 1.17a should also refer to the last statement of Theorem 1.16, to know that the Cartan involution $\mathrm{ad}\, u(-1)$ is a conjugate of the Cartan involution 1, and hence equal to 1.
- p45, Proposition 4.8: In the proof of Theorem 1.21, G was adjoint, so it would be better to change “$G(\mathbb{R})^+ \to \mathrm{Hol}(D)^+$” to “$G(\mathbb{R})^+ \to G^{\mathrm{ad}}(\mathbb{R})^+ \to \mathrm{Hol}(D)^+$”.
- p45, first sentence of last paragraph: $G_{\mathbb{R}}$ should be $G^{\mathrm{ad}}_{\mathbb{R}}$.
- p46, last sentence of Lemma 4.12: “such” should be “such that”.
- p46: Lemma 4.12 is never mentioned again after it is proved. What is its purpose?
- p47, first sentence of Remark 4.13: Perhaps give $\mathrm{SL}_2(\mathbb{Q}) \to \mathrm{PGL}_2(\mathbb{Q})$ as an example.
- p47, Example 4.14(b): Is it possible to say more explicitly what the congruence subgroups of $\mathrm{PGL}_2(\mathbb{Q})$ are?
- p48, Remark 4.17: Perhaps to emphasize what each example is illustrating, write “$\mathbb{G}_m$ (not semisimple)” in (a), and “PGL_2 (not simply connected)” in (b).
- p48, Proposition 4.18: In order to make sense of this, one needs to know that $G(\mathbb{Q})$ acts on D, i.e., that $G(\mathbb{Q})$ maps into $G^{\mathrm{ad}}(\mathbb{R})^+$. For this, it seems that one needs Theorem 5.2, which says that $G(\mathbb{R})$ is connected.
- p48, (28): It would help to have parentheses around $D \times G(\mathbb{A}_f)$, or else to mention a convention that $\times$ takes precedence over $\backslash$ and $/$.

- p49, Lemma 4.20: Is the word "separated" being applied to one space at a time (to mean Hausdorff) or to a pair of spaces (to mean that there are disjoint open sets containing them)? Both meanings are standard, so the word is ambiguous.
- p49, Lemma 4.20: The injectivity statement is wrong. For example, if $G = \mathbb{Z}_p$ acts on $X = \mathbb{Z}_p/\mathbb{Z}$ by translation, and $G_i = p^i\mathbb{Z}_p$, then $X/\bigcap G_i = \mathbb{Z}_p/\mathbb{Z}$, but $X/G_i = \{0\}$ for all i. If G and X are both Hausdorff, then injectivity and surjectivity both hold without any further assumption on separating orbits.
- p50, SV2: G^{ad} should be $G^{\text{ad}}_{\mathbb{R}}$.
- p52, last line of proof of Corollary 5.3: Insert "Gal($\mathbb{C}/\mathbb{R}$) is finite and" before "N is finite".
- p54, line 2: $G(\mathbb{R})$ should be $G_{\mathbb{R}}$.
- p54, last sentence of proof of Proposition 5.9: Is it supposed to be obvious that SV1 implies that $(V, \rho \circ h)_h$ is a variation of hodge structures? It would help to have more explanation here.
- p56, footnote 42: Why is it wrong to say "associated to"?
- p57, line 4: Insert "to" after "canonically isomorphic".
- p57, Lemma 5.19: In the first sentence, remove the second H.
- p57, Lemma 5.19(a): "For every finite prime" should be "For every finite prime ℓ" so that the ℓ in $\mathbb{Q}_\ell$ is defined.
- p57, Lemma 5.19(a): Change = to "is" since the sentence is missing a verb.
- p58, second paragraph of the proof of Lemma 5.20: At the end of the sentence starting "Now…" one could add "by Lemma 5.18".
- p58, end of proof of Lemma 5.20: Add a period.
- p58, just after the proof of Lemma 5.20: One should write "Assume that G^{der} is simply connected." here since so far the scope of this assumption has been only inside the theorems and lemmas earlier in this section.
- p58, line after the last display: What is the meaning of the subscript in the notation $[1]_K$?
- p58, same sentence: insert "for" before "some".
- p58, three lines later: The der superscript near at the end of this line is smaller than the others.
- p58, footnote 44, first sentence: Since this is nonabelian cohomology, the description of the equivalence relation on crossed homomorphisms (1-cocycles) is not correct.
- p58, footnote 44: Galois cohomology has appeared earlier, in the proof of Corollary 5.3 and in Lemma 5.19, for example, so it is strange to be defining it at this point.
- p60, Remark 5.23: Is the second half of the display intended to be a definition of Γ? This is not the Γ on p58.
- p60, next line: Insert "the" before "quotient", and change "larger group than Γ" to "group larger than Γ".
- p60, last paragraph: If G is an algebraic group over $\mathbb{Q}$, it is not necessarily true that every homomorphism $\mathbb{G}_{m\mathbb{R}} \to G_{\mathbb{R}}$ is defined over $\mathbb{Q}^{\text{al}}$. Maybe it should be said that w_X is mapping into a torus defined over $\mathbb{Q}$?
- p61, footnote 46: Change "it" to "SV6" for clarity.
- p61, two sentences after the big display: To explain why (G, X) satisfies SV3, it would be helpful to mention that G^{ad} is $\mathbb{Q}$-simple.
- p61, second display: To motivate this example of a $T(\mathbb{Z})$, one could mention that $T(L) = \text{Hom}(X^*(T), L^\times)$ and $T(\mathbb{Q}) = \text{Hom}(X^*(T), L^\times)^{\text{Gal}(L/\mathbb{Q})}$.

- p61, footnote 48: It takes about as much space to write one down as it does to say that it is easy to write one down. So perhaps write "The open subgroup $(1+4\mathbb{Z}_2) \times \prod_{p \text{ odd}} \mathbb{Z}_p^\times$ of $\mathbb{A}_f^\times$ intersects $\mathbb{Q}^\times$ in $\{1\}$."
- p62, Remark 5.27: In "A real torus is anisotropic if and only if it is compact", change "it" to "its group of $\mathbb{R}$-points".
- p62, Remark 5.27: At the end of the "Note that" sentence, add the explanation "by the previous sentence applied to $T = Z^\circ$".
- p62, next sentence: Insert "a" before CM-field.
- p62, next sentence: Define ι to be complex conjugation on L.
- p63, second display: What is the definition of S/K?
- p66, exact sequence with $\mathrm{GSp}(\psi)$ in the middle: This is not exact when $\dim V = 0$. So either assume $\dim V > 0$, or redefine $\mathrm{GSp}(\psi)$ as the set of pairs $(g, \lambda) \in \mathrm{GL}(V) \times \mathbb{G}_m$ such that $\psi(gu, gv) = \lambda\psi(u, v)$ for all $u, v \in V$.
- p66, last line excluding footnotes: The quantification of z is missing. Presumably the claim is that for all $z \in \mathbb{C}^\times$, we have $h_J(z) \in G(\mathbb{R})$.
- p66, last line excluding footnotes: To make "see the dictionary" more precise, perhaps number the third display on p65 and refer to it.
- p67, line 3: In the definition of X^+, one should add "such that $\psi(Ju, Jv) = \psi(u, v)$ for all $u, v \in V(\mathbb{R})$".
- p67, verification of (SV1): $z/\overline{z}$ and $\overline{z}/z$ should be switched.
- p68, Exercise 6.2(b): g should be such that $\dim V = 2g$.
- p68, second condition in the definition of $\mathcal{H}_K$: For clarity, change $\pm s$ to "either s or $-s$".
- p68, third condition in the definition of $\mathcal{H}_K$: For clarity, change "sending ψ to" to "under which ψ corresponds to". The same comment applies in the line before the second display on p68.
- p70, proof of Proposition 6.5: "doubly periodic" makes sense when $n = 1$ (when there are two independent periods), but sounds strange here. Maybe just write "is a periodic function with respect to Λ" or just "is a periodic function".
- p71, paragraph before Theorem 6.8: The first part of the first sentence duplicates the second sentence in the last paragraph of p70.
- p71, first display of Remark 6.10: $V^{-1,0}$ should be $V^{0,-1}$.
- p72: The paragraph should begin with "Fix (V, ψ) as on p68." since V has been used to mean other things since then.
- p72, line 4: Presumably $V_f(\mathbb{A}_f)$ should be $V_A(\mathbb{A}_f)$ where $V_A := H_1(A, \mathbb{Q})$? Or write $H_1(A, \mathbb{Q}) \otimes \mathbb{A}_f$, or define and use the notation $V_f(A)$ used in later sections?
- p73, two lines after Definition 7.1: The v looks a lot like ν, so perhaps use q instead, on both sides of the equation?
- p73, two lines after Definition 7.1: $h \circ \nu$ should be $\nu \circ h$.
- p74: There is confusion here between the V of the second paragraph and the symplectic space V that appears in the definition of ηK in the definition of $\mathcal{M}_K$; maybe use a different letter? Also, there is confusion between the fixed A in the second paragraph and the variable A that appears in the definition of $\mathcal{M}_K$; again maybe use a different letter? Finally, $V_f(A)$ has not been defined.
- p75, end: Perhaps mention that this is called the Lefschetz $(1,1)$-theorem.
- p76, line 6: after "A is **simple** if" insert "it is nonzero and".

- p76, line 7: after $M_n(D)$ insert "with $n \geq 1$".
- p76, line 11: It would be better to use a letter other than n for the subscript in B_n since this n has nothing to do with the n of the previous paragraph, and it is also unrelated to the n in Proposition 8.3.
- p76, Proposition 8.3: It would be nice to mention the reason for the labels **(A)**, **(C)**, **(BD)**.
- p77, line 2: Instead of citing a reference, one could simply given the reason: "because the factors in any such product are the minimal nonzero 2-sided ideals of B".
- p77, line 9: It is not clear what "Relative to a suitable basis" is referring to. Perhaps instead write "Composing the isomorphism $B \xrightarrow{\sim} M_n(k)$ with conjugation-by-g for some $g \in \mathrm{GL}_n(k)$ changes u to gug^t, so we may assume that u is I or J, ...".
- p77, line 12: It is not clear why the Noether–Skolem theorem is relevant. Anyway, isn't the conclusion obvious?
- p77, (35): Change the comma to " and ".
- p77, four lines after (35): The notation $\mathrm{GL}(V)$ is ambiguous. Clarify that here V is being considered as a vector space over k.
- p77, display after (35): $G(\mathbb{Q})$ should be $G(k)$.
- p77, display after (35): Insert "for" before "some".
- p77, Example 8.5: Before "and let $B =$", insert "let W be a finite free F-module" (since W here is not the W of Proposition 8.4).
- p77, footnote 61: Change "in one variable" to "the first variable".
- p77, end (excluding footnote): Perhaps add ", so $\phi(bw, w') = \phi(w, b^*w')$ for all $b \in B$ and $w, w' \in W$."
- p78, line 1: It would be natural to define "skew-hermitian form" here, given that the easier term "hermitian form" is defined on p79.
- p78, sentence containing (37): For clarity, after "the involution $*$" insert "of $\mathrm{End}_F V$".
- p78, (37) and (38): It would be better to write c^*c instead of cc^*. (Both are correct, but the former is what comes out naturally from the definitions. Also, the former is used in Proposition 8.10.)
- p78, Example 8.6: Insert "Let W be a finite-dimensional k-vector space." before "Let $B = \cdots$".
- p78, two lines after the last display: For clarity, after "the involution $*$" insert "of $\mathrm{End}_k V$".
- p78, end of Example 8.6: What is "(cf. 8.5)" trying to communicate? Only that a similar argument to that in Example 8.5 is needed?
- p79, last sentence of Remark 8.8: $\mathbb{Q}$ should be k in four places.
- p79, definition of **hermitian form**: For clarity, change "bilinear" to "$\mathbb{R}$-bilinear".
- p79, footnote 63: Change "in one of the variables" to "the first variable".
- p79, proof of Proposition 8.10(b)$\Rightarrow$(c): For clarity, change "basis" to "$\mathbb{R}$-basis".
- p80, display after (39): Perhaps mention somewhere that the r is not a typo, but the index of the coordinate.
- p81, Theorem 8.17, definition of ηK: H^1 should be H_1.
- p82, Proposition 8.19: "Aeven" looks funny. Perhaps make "even" a subscript?
- p82, first sentence of proof of Lemma 8.21: Use larger outer parentheses for the kernel since otherwise it is easy to misread this.
- p84, last display: e^t has not been defined.

- p84, last sentence: Previously, on p28, “polarization” was defined only for hodge structures of a fixed weight. Thus it is not clear what “polarizable” means here. Does it mean that each component of the weight decomposition is polarizable? Why does this follow from the polarizability of $H^*(V, \mathbb{Q})$?
- p85, Proposition 9.1: The proof does not seem to follow immediately from the definitions. For example, I suppose that one must use the identity $\mathbb{Q}(1) = \bigwedge^2 H_1(E, \mathbb{Q})$ for an elliptic curve E. Also one must observe that subquotients are the same as direct summands because of the polarization.
- p85, Proposition 9.3: Twice in the statement and once in the proof, $h \circ \rho$ should be $\rho \circ h$.
- p85, paragraph before Theorem 9.4: In (a), what is the exponent m? There does not seem to be a definition of m.
- p85, bottom: If a definition of $V_f(A)$ is not added to Sections 6 or 7, then it should be added here.
- p87, line 9: $h \circ \rho_{\mathbb{R}}$ should be $\rho_{\mathbb{R}} \circ h$.
- p88, end of first line of the first paragraph: Change $\in$ to “is in” so that the sentence has a verb.
- p.88, second paragraph: For make the meaning clearer, consider changing “big subgroup” to “finite-index subgroup”.
- p89, definition of **characteristic polynomial**: Perhaps change $\deg(a - n)$ to $\deg(n - a)$. They are equal, but it is more obvious that the latter is monic.
- p90, line 1: after “it” insert “is nonzero and”.
- p90, definition of **CM-type** abelian variety: It would be simpler, more natural, and more useful to define it by the condition that $\mathrm{Tgt}_0(A) \simeq \mathbb{C}^{\Phi}$ as $(E \otimes \mathbb{C})$-module, where Φ is defined as on p91. (Then (43) is an immediate consequence.)
- Remark 10.1(b): One could give an example: If B is an elliptic curve with CM by an order in K, then any degree 2 extension E of K embeds in $M_2(K) \simeq \mathrm{End}^0(B \times B)$, but such fields E need not be CM-fields.
- p91, middle: For clarity, change “remaining properties” to “remaining conditions for ψ to be a riemann form”.
- p91, 11th line from the bottom: muliplication should be multiplication.
- p91, two lines above the last display: It is not necessarily true that $\mathbb{Q}\Lambda = \Phi(E)$. It should say “$\mathbb{Q}\Lambda = \lambda\Phi(E)$ for some $\lambda \in (E \otimes \mathbb{R})^{\times}$”. The rest of the proof needs to be adjusted accordingly.
- p92, second to last sentence of the proof of Proposition 10.3: What is the intended argument that the specialization has the same CM-type? One way to explain it would be to say that the CM-type is determined by the set of eigenvalues of a generator e of E over $\mathbb{Q}$ acting on the tangent space, and this set is unchanged by the change of ground ring from $\mathbb{C}$ to R to $\mathbb{Q}^{\mathrm{al}}$.
- p92: The first paragraph after Remark 10.4 should be moved before Proposition 10.3, since the proof of Proposition 10.3 uses the notion of CM-type for abelian varieties over the subfield $\mathbb{Q}^{\mathrm{al}}$ of $\mathbb{C}$.
- p92, last paragraph: The notation $\mathcal{O}_{K,\mathfrak{P}}$ has not been defined. Does this denote the localization or its completion?
- p92, six lines from the bottom (excluding the footnote): “homogeneous $\bar{\mathfrak{a}}$ ideal” should be “homogeneous ideal $\bar{\mathfrak{a}}$”.

- p92, two lines from the bottom (excluding the footnote): "homomorphism" is misspelled.
- p93, last display: $a_i \in \mathbb{F}$ is not a consequence, so it should be on the left side of the implication.
- p93, Proposition 10.6(b): The first strange symbol should be $\pi_{\mathbb{A}^n}$.
- p93, Proposition 10.6(b): It is unclear whether (b) is being required for all $n \geq 0$ or just one positive integer n. In fact, it would suffice to have (b) for $\mathbb{A}^1$; it might be preferable to state it that way.
- p94, first display: To clarify, insert $= \rho(q)$ between q and $\rho(\pi) \cdot \rho(q/\pi)$.
- p94, line after the first display: The strange symbol should again be π.
- p94, Theorem 10.10: One could change "contains all conjugates of E" to "contains E" since k is already galois over $\mathbb{Q}$.
- p94, Theorem 10.10: Perhaps the definition of $|S|$ could go into the "Notation and conventions" section on p6?
- p94, immediately after Theorem 10.10: It would be nice to mention here that a sketch of a proof of Theorem 10.10 will be given at the end of this section.
- p95, second sentence of the "The $\mathcal{O}_E$-structure of the tangent space" subsection: Before the comma, insert "and $r_i \geq 1$".
- p95, same sentence: Change "the set of pairs" to "the multiset of pairs".
- p96, line before (48): To clarify, change "induces" to "restricts to".
- p96, (48): Add a period at the end.
- p96, sentence including (48): Why is this true?
- p96, display before (49): On the left side, the subscript $\mathcal{O}_E$ should be $\varphi\mathcal{O}_E$.
- p97, middle: Property (a) seems unnecessary. It (with the set of ramified places replaced by some other finite set of places) is implied by the continuity of rec_E.
- p98, line after (50): To prevent misunderstanding, change "with" to "up to".
- p98, second to last line of footnote 72: After "representation" insert "of".
- p98, last line of footnote 72: Since $(\mathbb{G}_m)_{E/\mathbb{Q}}$ is a group scheme, and E is a field, the usage of "extends" here is slightly weird. For clarity, perhaps end the sentence at the comma before which, and start a new sentence saying "The action of $E^\times$ on V extends to an action of $E\ldots$".
- p99, Theorem 11.2: The notion of E-linear isogeny has not been defined. This might be confusing, since it seems that α is not an isogeny at all, but rather an isomorphism in the category AV^0 that respects the E-actions (which also exist only in that category).
- p100, end of the existence proof in the proof of Theorem 11.2: The α chosen earlier in the proof on p99 might not satisfy the condition of Theorem 11.2. The correct α will depend on s, so α needs to be adjusted.
- p102, footnote 74: Here is a more conceptual proof: T centralizes $\mu(\mathbb{G}_m)$ and $\mu'(\mathbb{G}_m)$, the latter implies that $\mathrm{ad}(g)(T)$ centralizes $\mathrm{ad}(g)\mu'(\mathbb{G}_m) = \mu(\mathbb{G}_m)$.
- p102, Definition 12.2: For psychological reasons, it would be nice to say right away that "We will see in Example 12.4(b) that this generalizes the notion of reflex field of a CM-type."
- p102, sentence after Definition 12.2: After "reflex field" insert "is".
- p102, sentence after Definition 12.2: This is true, but why not say instead that it is a subfield of $\mathbb{Q}^{\mathrm{al}}$?

- p102, second and third sentences of Remark 12.3(c): The following might be an easier-to-digest explanation, even if the content is roughly the same: "Then i induces a $\mathrm{Gal}(\mathbb{Q}^{\mathrm{al}}/\mathbb{Q})$-equivariant map $\mathcal{C}_G(\mathbb{Q}^{\mathrm{al}}) \to \mathcal{C}_{G'}(\mathbb{Q}^{\mathrm{al}})$ sending $c(X)$ to $c(X')$." (If this is used, then in the sentence on p101 introducing $\mathcal{C}(k)$, write "…we write $\mathcal{C}(k) = \mathcal{C}_G(k)$…", so that the use of subscripts on the $\mathcal{C}$ is understood.)
- p102, Example 12.4(a): $E(T, h)$ should be $E(T, \{h\})$.
- p103, top: It is not explained how the formula for h_Φ leads to the formula for μ_Φ. Perhaps it would be clearer to say "Define $h_\Phi\colon \mathbb{S} \to T_\mathbb{R}$ so that $\mathbb{C}^\times \times \mathbb{C}^\times = \mathbb{S}(\mathbb{C}) \longrightarrow T(\mathbb{C}) = (\mathbb{C}^\Phi)^\times \times (\mathbb{C}^{\overline{\Phi}})^\times$ is $(z_1, z_2) \mapsto ((z_1, \ldots, z_1), (z_2, \ldots, z_2))$." This not only makes it clear what μ_Φ is, but also renders unnecessary the explicit description of $T(\mathbb{R})$ at the bottom of p102.
- p103, Example 12.4(d): The notation I_c has not been defined.
- p103, Example 12.4(d): How was the expression for $\mu(z)$ determined?
- p103, Example 12.4(e): Why does the galois group act on Δ?
- p103, line before the last display: Change "contains a μ" to "contains a unique μ" so that it is clear that the map in the display is well-defined.
- p103, footnote 76: "Serre group" and "CM-motive" have not been defined.
- p103, Remark 12.6: Between the first two sentences it would be helpful to say "Then $h(\mathbb{C}^\times)$ is contained in the centralizer of $T(\mathbb{R})$ in $G(\mathbb{R})$."
- p103, last sentence (excluding the footnote): This sentence is true even if T is not maximal, so it should be appear before the rest of Remark 12.6.
- p104, Example 12.7: The claim that $(\mathbb{G}_m)_{E/\mathbb{Q}}(\mathbb{R})$ fixes z is wrong; instead it fixes $-z$. This can be seen as follows:

  Since E has basis $\{1, z\}$ over $\mathbb{Q}$, the 2-dimensional $\mathbb{C}$-vector space $E \otimes \mathbb{C}$ has basis $1 \otimes 1$ and $z \otimes 1$. The map $E \otimes \mathbb{C} \to \mathbb{C}$ sending $e \otimes c$ to ec has 1-dimensional kernel spanned by $1 \otimes (-z) + z \otimes 1$, which with respect to our basis is $\begin{pmatrix} -z \\ 1 \end{pmatrix}$, which represents the point $-z \in \mathcal{H}_1^\pm$. This map is $(E \otimes \mathbb{R})$-linear, so $(E \otimes \mathbb{R})^\times$ fixes the kernel. That is, $(\mathbb{G}_m)_{E/\mathbb{Q}}(\mathbb{R})$ fixes $-z \in \mathcal{H}_1^\pm$.
- p104, Definition 12.8: Although the notation $[x, a]$ has been used before, the notation $[x, a]_K$ has not. In any case, the last time such notation appeared was many sections ago, so it would be nice to recall its meaning by replacing the first appearance of "$[x, a]_K$" by "the point $[x, a] \in G(\mathbb{Q})\backslash X \times G(\mathbb{A}_f)/K = \mathrm{Sh}_K(G, X)$".
- p104, Definition 12.8: It would be nice to explain the motivation for this definition. Is it inspired by Theorem 11.2? If so, one could at least say this.
- p105, second to last line, "To see this": What does "this" refer to?
- p105, same sentence: Change "one-dimensional E-vector space" to "one-dimensional E-vector space viewed as a $\mathbb{Q}$-vector space", so that in the next sentence, $\mathrm{GL}(V)$ is the automorphism group of a vector space over $\mathbb{Q}$ as it should be.
- p106, line 1: "E-homomorphism" should be "E-isomorphism".
- p106: Throughout, T^E should be T, to match the notation on p105.
- p106: At the end of the paragraph defining an action of $\mathrm{Gal}(\mathbb{Q}^{\mathrm{al}}/E^*)$ on $\mathcal{M}_K$ (the paragraph containing (55)), add "This defines a model of $\mathrm{Sh}_K(T, h_\Phi)$. Proposition 12.11 below will show that it is a canonical model." since presumably this is the point.
- p106, third line of the proof of Proposition 12.11: Change "isomorphism" to "E-linear isogeny".

- p106, proof of Proposition 12.11: The superscript $=1$ should be -1 (twice).
- p106, proof of Proposition 12.11: The notations $H_1(\alpha)$ and $V_f(\alpha)$ are used interchangeably; it would be best to write $V_f(\alpha)$ consistently (change it in two places).
- p107, footnote 79: Does "the second statement" refer to the claim that T_0 will become conjugate to T over $\mathbb{R}$?
- p107, last display: Insert "for" before "some".
- p108, proof of Lemma 13.5: The space X is not a variety, so it does not have a zariski topology. After saying that $G(\mathbb{Q})x$ is dense in X, the proof should read as follows:

  Then $G(\mathbb{Q})x \times G(\mathbb{A}_f)$ is dense in $X \times G(\mathbb{A}_f)$, and its image in $\mathrm{Sh}_K(G,X)(\mathbb{C})$ is dense for the complex topology, and a fortiori for the zariski topology. That image equals $\{[x,a]_K \mid a \in G(\mathbb{A}_f)\}$, since $[gx,b]_K = [x, g^{-1}b]$ for any $g \in G(\mathbb{Q})$ and $b \in G(\mathbb{A}_f)$.
- p108, second display: Explain that Sh_K means $\mathrm{Sh}_K(G,X)$.
- p108, end of the sentence before Theorem 13.6: Say that $\mathcal{T}(g)$ is a morphism of varieties, ultimately because of Theorem 3.14.
- p108: The paragraph after the proof of Theorem 13.7 really belongs in the proof of Theorem 13.7(a).
- p109, (56): From the wording it is not clear whether this is a statement whose proof is being omitted, or whether it is being claimed that this has already been proved.
- p109, last sentence: Delete "the" before "Zorn's lemma".
- p111, Proposition 14.7: The finitely generated $\mathbb{Z}$-modules in the second category should be free, to match the given definition of local system of $\mathbb{Z}$-modules.
- p112, definition of **variation of integral hodge structures on** S: An alternative to passing to the universal cover would be to define it locally (on a covering of S by open sets on which F becomes trivial). Perhaps this is slightly more natural?
- p112, definition of **variation of integral hodge structures on** S: On p29, "variation of hodge structure" was defined only when the base was a connected complex manifold, but here the notion is being applied to $\widetilde{S}$, which is not necessarily a complex manifold.
- p113, Corollary 14.11: It was said on p112 that $\mathrm{Sh}_K(G,X)$ would be abbreviated to Sh_K, so perhaps write the latter here.
- p113, Corollary 14.11: "A is of CM-type" should say "A is CM" to match Definition 14.9.
- p113, two lines before Proposition 14.12: $H_1(A,\mathbb{Q})$ should be $H_1(\sigma A,\mathbb{Q})$.
- p114, end of first sentence: Add ", which we denote by $\cdot$." This will help clarify the difference between σP and $\sigma \cdot P$ later on.
- p114-115: Throughout this "outline", consider using the abbreviation Sh_K consistently to reduce clutter (since the G and X are not changing here). The same could be done in the "Simple PEL Shimura varieties of type A or C" section.
- p114, "Define $\mathcal{H}_K^{\varepsilon}$ similarly." It is not clear what the definition is. Was $\mathcal{H}_K$ defined somewhere earlier?
- p114, next sentence: Does the $\mathrm{Aut}(\mathbb{C})$-equivariance of $\mathcal{M}_K \to \mathbb{Q}_{>0}\backslash\mathbb{A}_f^{\times}/\nu(K)$ come from the Weil pairing? It would be nice to say more here.
- p114, second display: The quantification of α and ζ is unclear. Presumably what is intended is to say "for every ζ that is a root of 1, and for every $\alpha \in \mathbb{A}_f^{\times}$".
- p114, last diagram: The α here seems to be different from the α in the previous display, but it is not defined, and not mentioned in the rest of the proof.

- p114, last (partial) paragraph: Presumably Λ is a $\mathbb{Z}$-lattice inside the $\mathbb{Q}$-vector space V? Why does there exist a Λ that is Γ_ε-invariant?
- p114, next sentence: Why is M a polarized integral variation of hodge structures?
- p115, line 3: After "can" insert "be".
- p115, line 5, "the map $u \mapsto x$": Is this α?
- p115, line 9: For clarity, after "fixes" insert "all".
- p116, Proposition 14.16(b): "isogeny of connected Shimura data" has not been defined.
- p116, paragraph before Remark 14.17: Is "primitive type" the same as what was called "primitive abelian type" in Definition 9.2(a)?
- p116, same sentence: The logic is not clear. Does one also need to prove an analogue for connected Shimura data of Proposition 14.14?
- p117, line 15: Delete "and" before "such that".
- p117, line 16: F should be F'.
- p118, definition of **simple**: Insert "is nonzero and" before "has".
- p118, next paragraph: It might be better to define re before this notation is used.
- p118, definitions of **algebra**, **central**, **division algebra**, **simple**: It is strange to say "Recall the definitions" given that these are not the standard definitions. (Usually F-algebras are not required to be finite-dimensional, and the homomorphism from $F \to A$ is not required to be injective. Also, the zero algebra should be considered to a central F-algebra, but not a division algebra, and not simple.)
- p119, Example 15.1(c): $x \mapsto x^p$ should be $x \mapsto x^q$, where q is the size of the residue field of F.
- p119, display in Example 15.1(c): It might help to say what a is here.
- p119, display in Example 15.1(c): Insert "for all" before "$z \in L$".
- p119, line after third display: Change "obviously" to "obviously associative".
- p120, second paragraph: It is not clear what the hypothesis on F is at this point.
- p120, line 8: Change "central simple division" to "central division algebra".
- p120, Example 15.3: The wording "E^λ can be taken to be..." makes it sound as if E^λ has been mentioned previously, but it has not been.
- p120, same line: $\mathbb{Q}_p/(T^r - p^s)$ makes no sense.
- p121, line 1: π_A has not been mentioned since Section 10, so it would be good to recall what this notation means.
- p121, paragraph before Theorem 15.5: Insert "simple" before "abelian variety".
- p122, line before (58): homomorphism should be homomorphism.
- p123, line 6: The equality $e_\sigma g e_\sigma^{-1} = \sigma g$ may be up to conjugation by $G(L)$.
- p123, second paragraph: What is the reason for the term **affine**? I.e., what relation does it have to other uses of the word "affine" in mathematics?
- p123, fourth display: The left hand side should probably be $e_{\sigma\tau}$.
- p123, definition of **homomorphism**: If L is finite, then the homomorphism of algebraic groups $G_1 \to G_2$ (over L) is not uniquely determined by its action on L-points, and it should be part of the data defining a homomorphisms of affine extensions. Otherwise Remark 15.8 fails. Probably the homomorphism of algebraic groups should be part of the data
- p123, Remark 15.8: It would be clearer if $H^1(F, \mathrm{GL}(V))$ were changed to $H^1(L/F, \mathrm{GL}(V))$.

- p123, Proposition 15.9: Presumably D is being *defined* as $\mathrm{End}(V_{\Gamma\chi})$. Actually, why introduce D at all if it is never used? Another reason not to give it the name D is that D means something very different later in this section.
- p124, line 5: Direct limits of free modules are not necessarily free. One could replace "free" by "flat".
- p124, fourth line from the bottom: $\mathbb{Q}_p^{\mathrm{un}\times}/\mathbb{Q}_p$ should be $\mathbb{Q}_p^{\mathrm{un}}/\mathbb{Q}_p$.
- p124, second line from the bottom, "the image": under what map? To what sets do $(1, a)$ and (F, σ) belong?
- p125, line 1, $\sigma^i \mapsto a^i$: Is this for $0 \leq i \leq n-1$? Otherwise it seems not to be well defined.
- p125, definition of W: It might help to specify the maps in this directed system.
- p125, first display: Say that v is a place of $\mathbb{Q}\{\pi\}$ above p.
- p125, first display: Define q.
- p125, Theorem 15.12: $\left(\frac{1}{2}\right)^{wt(\pi)}$ should be $\mathrm{wt}(\pi)/2$.
- p126, line 1: $\mathfrak{P}_l$ should be $\mathfrak{P}(l)$.
- p126, line 1: Is there a distinction being made between l and ℓ? Maybe l can equal p, but ℓ cannot? If this is so, it would be good to mention explicitly that l and ℓ will play these differing roles.
- p126, proof of Proposition 15.13: It would help to explain in more detail how to pass from ρ to $\rho(\ell)$.
- p126, last paragraph, "lattices": Presumably these are $\mathbb{Z}_l$-lattices? Of course, every finite-dimensional $\mathbb{Q}_l$-vector space contains a $\mathbb{Z}_l$-lattice. What properties are these lattices supposed to have? Why does this depend on ζ_ℓ
- p127, proof of Proposition 15.14(a): Where was this fact proved? Or why is it true?
- p127, two lines after the diagram, "the argument": Which argument is being referred to? To a reader without the TEX file, it is not clear what is a subsection and what is a subsubsection.
- p127, line before Proposition 15.15, "the answer is yes": Isn't this obvious from the fact implicit at the bottom of p125 that the abelian category of fake abelian varieties is equivalent to the abelian category of abelian varieties up to isogeny?
- p128, line 7: realizations should be realization.
- p129, line 6, "When we have a description... the same description holds over $\mathbb{Q}^{\mathrm{al}}$": Is this a theorem with a precise meaning or is it a heuristic principle?
- p129, "This is so when condition SV5 holds, but not otherwise": Why?
- p130, line 3, "bad reduction at the others": This seems wrong. For instance if $N < 11$, then the (compactified) quotient is $\mathbb{P}^1_{\mathbb{Q}}$, which has good reduction everywhere.
- p130, Example 16.2: What is wrong with $p = 2$?
- p130, last line of the first section: Has $\mathbb{A}_f^p$ been defined?
- p130, Theorem 16.4: It would be preferable to define "canonical good reduction" before it is used here.
- p131, line after second display: Should "étale" be "finite étale"? Otherwise the conditions seem rather weak: for instance, as one goes up any tower of curves over $k(\mathfrak{p})$, one could remove all ramification points at each level to obtain a tower of affine curves with étale maps. Maybe the condition that $G(\mathbb{A}_f^p)$ rules this out, however.

- p131, (b), blowing up a point in the closed fibre: This sounds wrong. If one blows up a smooth $\hat{\mathcal{O}}_{\mathfrak{p}}$-scheme at a closed point in its closed fibre, the total space is regular, but the new closed fibre is no longer smooth.
- p131, (61): To be more precise, the natural map from left to right is an isomorphism.
- p132, line 2: The definitions of $X^p(x)$ and $X_p(x)$ look strange since they appear not to depend on x.
- p132, third display: Insert "for" before "all $p \in \mathfrak{P}$".
- p132, last sentence of the "Definition of the group $I(\phi)$" section: To help the reader keep track of things, perhaps change "$\rho \circ \phi$ is a fake motive" to "$\rho \circ \phi \colon \mathfrak{P} \to E_V$ is a fake motive".
- p132, last line: $\zeta_\ell \circ \phi(\ell)$ should be $\phi(\ell) \circ \zeta_\ell$.
- p133, line 11: Insert "be the" before "Frobenius automorphism".
- p133, line 16: How is σ an element of $\mathfrak{P}(p)^{\mathrm{un}}$?
- p133, line 17: Why is there more than one possible $b(\phi)$? Where was there a choice?
- p133, line 19: $D(\phi \circ \rho)$ should be $D(\rho \circ \phi)$.
- p134, proof of Lemma 16.6: Was this fact mentioned earlier? Why is it true?
- p134, Lemma 16.7: $E_\infty \to \mathfrak{P}(\infty)$ should be $E_\infty \to E_G(\infty)$.
- p134, line before "**The condition at** $\ell \neq p$": $\zeta_\infty \circ \phi(\infty)$ should be $\phi(\infty) \circ \zeta_\infty$.
- p134, second line of the section "**The condition at** $\ell \neq p$": $\zeta_\ell \circ \phi(\ell)$ should be $\phi(\ell) \circ \zeta_\ell$.
- p134, second to last line: homomorphism should be homomorphisms.
- p135, last diagram: It would be better to put bars over each Sh.
- p136, first bullet in the definition of triple: What does "anisotropic modulo the centre of $G_\mathbb{R}$" mean if that centre is not contained in the torus? Does it mean that the quotient of the torus by its intersection with the centre of $G_\mathbb{R}$ is anisotropic?
- p136, first display: The "$\mathcal{N} \circ$" on the left side looks weird; is this what was intended? In any case, I think this definition is never used.
- p136, definition of I_0: Why is I_0 connected and reductive?
- p136, last paragraph: "stably conjugate" has not been defined. Does it mean that γ_0 and γ_ℓ are conjugate in $G(\mathbb{Q}_\ell^{\mathrm{al}})$?
- p136, fourth to last line: The isomorphisms j_ℓ and a are defined over fields neither of which is contained in the other. Is $j_\ell \circ a$ to be viewed as an isomorphism over $\mathbb{Q}_\ell^{\mathrm{al}}$?
- p136, third to last line: To clarify, after "inner automorphism" add "of $(I_0)_{\mathbb{Q}_\ell^{\mathrm{al}}}$".
- p136, second to last line: It seems to be being claimed that the I and a are the same. Why is this so?
- p137, line 4: Why is the resulting measure unchanged if (j_ℓ) is modified by an element of $I^{\mathrm{ad}}(\mathbb{A})$?
- p137, middle: In the definition of $I(\gamma_0; \gamma, \delta)$, ϕ_r has not been defined.
- p137, definition of **admissible pair**: $I_\phi(\mathbb{Q})$ has not been defined.
- p137, paragraph defining **admissible pair**: Presumably every γ in this paragraph should be γ_0, and γ' should be γ_0'.
- p138, line 2: It would be clearer to say "For a γ_0 belonging to a triple" since what is being defined is $c(\gamma_0)$, not c of a triple.
- p138, line 4: Why is this kernel finite?
- p139, end of first paragraph: Add "when $k = \mathbb{Q}$".
- p140, line 2: The logic of this sentence could be made clearer. Is the argument as follows?

Since G' is unirational, there exists a nonnegative integer n, a nonempty open subset U of $\mathbb{P}^n$, and a dominant morphism $f\colon U \to G'$. We may shrink U to assume that f is smooth, so that the induced map $U(\mathbb{R}) \to G'(\mathbb{R})$ is open. Since $\mathbb{P}^n(\mathbb{Q})$ is dense in $\mathbb{P}^n(\mathbb{R})$, we have that $U(\mathbb{Q})$ is dense in $U(\mathbb{R})$. Then $G'(\mathbb{Q})$ contains $f(U(\mathbb{Q}))$, which is dense in the nonempty open subset $f(U(\mathbb{R}))$ of $G'(\mathbb{R})$. Since the closure of $G'(\mathbb{Q})$ in $G'(\mathbb{R})$ is a group, it is open. But $G'(\mathbb{R})$ is connected, so this closure must be all of $G'(\mathbb{R})$.

Perhaps it would be better to make it a lemma (that if G is a unirational algebraic group over $\mathbb{Q}$ such that $G(\mathbb{R})$ is connected, then $G(\mathbb{Q})$ is dense in $G(\mathbb{R})$).

- p140, line 3: Why is $G'(\mathbb{R})$ connected?
- p140, end of first proof: The proof is incomplete, since G in Theorem 5.4 is not necessarily reductive.
- p140, Second Proof: "Sept 1" should be "Sept. 1".
- p140, line 3 of Second Proof: "no 27" should be "no. 27".
- p140, Second Proof: Perhaps add a footnote to define "regular".
- p140, Second Proof, third to last line: "a torus defined over $\mathbb{Q}$" should be "a torus T defined over $\mathbb{Q}$" so that T is defined when it appears later in this line.
- p140, Second Proof, second to last line: Insert "torus" after "maximal $\mathbb{R}$-split".
- p140, third to last line: Why does the isogeny extend to a homomorphism of group schemes over $\operatorname{Spec}(\mathbb{Z})$? Is this a general property of homomorphisms of algebraic groups over $\mathbb{Q}$?
- p140, last line: Why is $Z(\mathbb{Z}_\ell) \times G^{\mathrm{der}}(\mathbb{Z}_\ell) \to G(\mathbb{Z}_\ell)$ surjective?

Department of Mathematics, Massachusetts Institute of Technology, Cambridge, MA 02139-4307, USA

E-mail address: poonen@math.mit.edu

URL: http://math.mit.edu/~poonen/