The Work of John Tate

J.S. Milne

Tate helped shape the great reformulation of arithmetic and geometry which has taken place since the 1950s.

Andrew Wiles.[1]

This is an exposition of Tate's work, written on the occasion of the award to him of the Abel prize. True to the epigraph, I have attempted to explain it in the context of the "great reformulation".

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[1] Introduction to Tate's talk at the conference on the Millenium Prizes, 2000.

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